542. Geocentric Parallax

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MATHEMATICAL NOTES.

542. [U.] *Geocentric Parallax.*

[The following elementary method of obtaining the fundamental equations of geocentric parallax is directly by Spherical Trigonometry, and may be of interest to readers. The notation here followed is the same as that in Art. **93** of Sir Robert Ball's *Spherical Astronomy.*]

Let $Z(\vartheta, \phi')$, $S(\alpha, \delta)$, $S'(\alpha', \delta')$ be the points on the celestial sphere (Fig 2) to which the lines OO', OS, $O'S$ (Fig. 1) are severally directed ; ρ the earth's

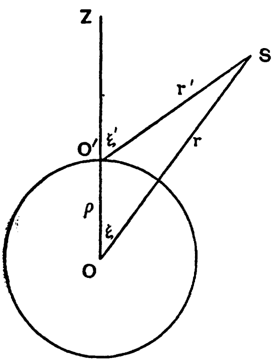

FIG. 1.

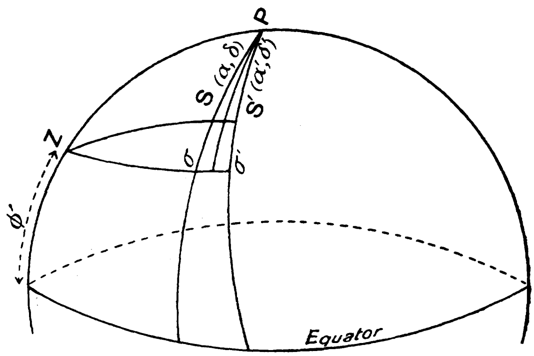

FIG. 2.

radius and r, r' the distances of the celestial body S from the earth's centre O, and the observer's position O', respectively ; P the pole of the celestial sphere.

To find $\alpha' - \alpha$, the parallax in Right Ascension.

Let $$P\hat{S'}Z = \epsilon, \quad Z\hat{O}S = \xi, \quad Z\hat{O'}S = \xi'.$$

From the triangle $O'OS$ in Fig. 1, we have

$$\frac{\sin SS'}{\rho} = \frac{\sin S'Z}{r} = \frac{\sin SZ}{r'} ; \qquad \text{.........(1)}$$

and from the spherical triangles PSS' and PZS', we get

$$\frac{\sin SS'}{\sin(\alpha' - \alpha)} = \frac{\cos\delta}{\sin\epsilon}, \qquad \text{.........(2)}$$

$$\frac{\sin ZS'}{\sin(\alpha' - \vartheta)} = \frac{\cos\phi'}{\sin\epsilon}. \qquad \text{.........(3)}$$

Dividing (2) by (3) and substituting for the arcs from (1), we get

$$\frac{\rho}{r} \cdot \frac{\sin(\alpha' - \vartheta)}{\sin(\alpha' - \alpha)} = \frac{\cos\delta}{\cos\phi'},$$

i.e. $$\frac{\rho}{r}\{\cos(\alpha' - \vartheta) + \cot(\alpha' - \alpha)\sin(\alpha - \vartheta)\} = \frac{\cos\delta}{\cos\phi'} ;$$

whence we derive the formula

$$\tan(\alpha' - \alpha) = -\frac{\frac{\rho}{r}\cos\phi'\sin(\vartheta - \alpha)}{\cos\delta - \frac{\rho}{r}\cos\phi'\cos(\vartheta - \alpha)}. \qquad \text{.........(A)}$$

To find $\delta' - \delta$, the parallax in Declination.

Draw $Z\sigma\sigma'$ perpendicular to the bisector of the angle SPS' to meet the declination circles of S, S' in σ, σ' respectively.

Evidently $$P\sigma = P\sigma'.$$

Let the declination of σ or σ' be γ.

From the usual relation between the arcs joining three points on a great circle and another point, we have

$$\cos PZ \sin SS' + \cos PS' \sin ZS = \cos PS \sin ZS',$$

$$i.e.\quad r \sin \delta - r' \sin \delta' = \rho\beta \sin \gamma, \text{ where } \beta = \frac{\sin \phi'}{\sin \gamma}. \quad \ldots\ldots\ldots\ldots(4)$$

Again, from the spherical triangles $Z\sigma S$ and $Z\sigma' S'$, we get

$$\frac{\sin \sigma S}{\sin ZS} = \frac{\sin \sigma ZS}{\sin P\sigma Z} = \frac{\sin \sigma' S'}{\sin ZS'},$$

$$i.e.\quad \frac{\sin(\delta - \gamma)}{\sin(\delta' - \gamma)} = \frac{r'}{r},$$

$$i.e.\quad \sin \gamma (r \cos \delta - r' \cos \delta') = \cos \gamma (r \sin \delta - r' \sin \delta'),$$

$$i.e.\quad r \cos \delta - r' \cos \delta' = \rho\beta \cos \gamma. \quad \ldots\ldots\ldots\ldots(5)$$

Multiplying (4) by $\cos \delta'$ and (5) by $-\sin \delta'$ and adding, we get

$$\frac{\sin(\gamma - \delta')}{\sin(\delta - \delta')} = \frac{r}{\rho\beta},$$

$$i.e.\quad \sin(\gamma - \delta) \cot(\delta - \delta') + \cos(\gamma - \delta) = \frac{r}{\rho\beta},$$

from which we easily derive

$$\tan(\delta' - \delta) = \frac{\rho\beta \sin(\delta - \gamma)}{r - \rho\beta \cos(\delta - \gamma)}. \quad \ldots\ldots\ldots\ldots(\text{B})$$

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543. [R. X. 4.] *A Graphical Treatment of Simple Harmonic Motion.*

The importance of a treatment of Simple Harmonic Motion in any course of Mechanics will be granted at once by any reader of this note. Yet, so far as the author is aware, one of two methods is usually adopted, each having, in his opinion, serious disadvantages. The first, and perhaps more common, method makes S.H.M. depend upon the previously considered properties of a uniform circular motion, of which it is the projection. The two main objections to this are, first, that the fact of the acceleration being proportional to the displacement emerges only after a time, instead of being considered as the fundamental fact determining the motion; secondly, that there is usually serious confusion when these ideas are applied to the pendulum, since its bob moves in an arc of a circle. The other method, which consists in integrating the differential equation of motion, while mathematically unexceptionable, demands a much better knowledge of the principles and processes of the Calculus than can be expected of boys reaching the subject for the first time, probably (and preferably) in a course having a distinct physical bias. These considerations have led to a third method, explained below

It is assumed that laboratory work will have supplied the main facts with regard to the simple pendulum, *i.e.* its isochronism, period $\propto \sqrt{\text{length}}$, restoring force $\propto$ displacement. (It is quite easy, and instructive, to show this last fact experimentally, afterwards pointing out its connection with the parallelogram of forces.) It is now necessary to show mathematically the interdependence of these facts.