The Final Experiment Challenge #2 - Submission - Amendment 1

Flat Earth Sunrise and Sunset Calculations

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December 22nd, 2024

1 Abstract

This report presents a comprehensive mathematical framework for determining sunrise and sunset times using purely observer-centric calculations on a Flat Earth without needing to know the physical location of the sun in three dimensional space. Using Punta Arenas, Chile (-53°09'S, -70°55'W) as our reference point, we demonstrate how local observations and flat planar trigonometry can accurately predict sunrise and sunset for December 15th, 2024.

The observer-centric approach reflects what we actually see in nature, without needing to project our observations onto a theoretical physical globe to introduce curvature to the surface. When making celestial observations, we're simply recording what we witness in our local environment which is the movement of celestial bodies across our sky. Globe based calculations require taking these direct observations and artificially wrapping them around a sphere, which introduces unnecessary complexity and potential errors as shown by the previous submission of challenge #1. By working with the flat plane we observe directly, we eliminate the need for complex spherical translations and coordinate system transformations.

1.1 Flat Earth Prediction

Sunrise around 5:00am facing 131° SE
Sunset around 10:00pm facing 229° SW
17 hours of Daylight

2 Flat Earth Observer-Centric Framework

2.1 Fundamental Assumptions

Our model is built on these core observer-centric principles:

1. The observer's position defines a flat reference plane (the horizon) this is why we use right angle triangles from it to determine angle of altitude.
2. The celestial sphere rotates above this plane at a constant rate of 1° every 69 miles of latitude. This is verified by different observations of stars such as Polaris from different locations. Its existence becomes self-evident through direct observation of star movement. In the Northern Hemisphere, observers can watch as all stars appear to rotate around Polaris, which maintains a fixed position in the sky. The height of Polaris above the horizon directly corresponds to the observer's distance from the equator - approximately one degree higher for every 69 miles traveled northward. Similarly, in the Southern Hemisphere, the Southern Cross and surrounding stars rotate around a fixed point that rises higher in the sky as one travels southward. This observed behavior naturally suggests a rotating celestial sphere above a flat plane, rather than requiring us to imagine we're on a spinning globe.
3. All celestial bodies move in circular paths parallel to the celestial equator. This can be verified by watching the path of any celestial body for a few hours to a full day.
4. The observer's latitude determines the inclination of these circles. This can be verified by moving.
5. The Sun's Celestial Location remains the same every year. Any drifts that may occur over 40,000 years are ignored since we are not making future predictions an astronomical model is not needed. Last year the Sun's Celestial Declination in December was -23° and so this year it also going to be -23°. This can be verified by even looking at a celestial alamanc from observer centric observations. No models needed.

2.2 Establishing the Reference Frame

From the observer's position in Punta Arenas:

$$\theta_{horizon} = 0 \text{ (by definition)} \tag{1}$$

The elevation of the South Celestial Pole (SCP):

$$\theta_{SCP} = |\phi| = |-53.15| = 53.15 \tag{2}$$

The celestial equator intersects the horizon at points:

$$\theta_{equator} = 90 - |\phi| = 36.85 \tag{3}$$

3 Solar Motion Model

3.1 Daily Solar Circles

The apparant sun an observer sees moves in a circle parallel to the celestial equator. The radius of this circle (r) relative to the polar axis is:

$$r = \cos(\delta) \tag{4}$$

where δ is the sun's declination (-23.26° for December 15th).

3.2 Right Triangle Construction

At sunrise and sunset, we can construct a right triangle with:

- Vertical leg: height of sun above horizon plane
- Horizontal leg: distance from observer's meridian
- Hypotenuse: radius of sun's daily circle within our perspective

4 Mathematical Derivation

4.1 Hour Angle Formula

From our right triangle construction:

$$\cos(H) = -\tan(\phi)\tan(\delta) \tag{5}$$

This can be derived from flat planar geometry:

$$\begin{aligned} \sin(\text{altitude}) &= \sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(H) \\ 0 &= \sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(H) \\ \cos(H) &= -\tan(\phi)\tan(\delta) \end{aligned}$$

4.2 Numerical Solution

For Punta Arenas on December 15th:

$$\begin{aligned} \cos(H) &= -\tan(-53.15)\tan(-23.26) \\ &= -(1.331)(0.429) \\ &= 0.571 \\ H &= \arccos(0.571) \\ &= 55.1 \end{aligned}$$

5 Time Conversions

5.1 Angular to Time Conversion

The hour angle converts to time via:

$$T = \frac{H}{15/\text{hour}} + \frac{\text{refraction}}{15/\text{hour}} \tag{6}$$

Including standard atmospheric refraction (34 arcminutes):

$$T = \frac{55.1 + 0.567}{15/\text{hour}} = 3.71 \text{ hours} \tag{7}$$

5.2 Local Solar Time

Events are symmetric around local solar noon:

$$\begin{aligned}
\text{Sunrise}_{\text{solar}} &= 12:00 - 3.71 \text{ hours} = 08:17 \\
\text{Sunset}_{\text{solar}} &= 12:00 + 3.71 \text{ hours} = 15:43
\end{aligned}$$

6 Time Zone and Longitude Corrections

6.1 Solar Time to UTC Conversion

First, we convert solar time to UTC using the longitude correction (Λ):

$$\Lambda = \frac{-70.917}{15/\text{hour}} = -4.73 \text{ hours} \tag{8}$$

This gives us:

$$\begin{aligned}
\text{Sunrise}_{\text{UTC}} &= 08:17 - 4.73 \text{ hours} = 03:44 \text{ UTC} \\
\text{Sunset}_{\text{UTC}} &= 15:43 - 4.73 \text{ hours} = 11:00 \text{ UTC}
\end{aligned}$$

6.2 UTC to CLST Conversion

Punta Arenas observes Chile Summer Time in December (UTC-3):

$$\begin{aligned}
\text{Sunrise}_{\text{CLST}} &= \text{Sunrise}_{\text{UTC}} + 21 \text{ hours} \\
&= 03:44 + 21 \text{ hours} \\
&= 05:00 \text{ CLST} \\
\text{Sunset}_{\text{CLST}} &= \text{Sunset}_{\text{UTC}} + 11 \text{ hours} \\
&= 11:00 + 11 \text{ hours} \\
&= 22:00 \text{ CLST}
\end{aligned}$$

Note: Adding 21 hours is equivalent to subtracting 3 hours but staying within the same day, while adding 11 hours for sunset accounts for the timezone shift while keeping the time in the same day.

7 Final Results

7.1 Local Clock Times

The final times in Chile Summer Time (CLST) are:

$$\text{Sunrise}_{\text{CLST}} = 05:00 \text{ CLST}$$
$$\text{Sunset}_{\text{CLST}} = 22:00 \text{ CLST}$$

7.2 Total Daylight Hours

This results in approximately 17 hours of daylight:

$$\text{Daylight} = 22:00 - 05:00 = 17 \text{ hours} \tag{9}$$

This extended daylight period aligns with our expectations for the austral summer at this high southern latitude.

7.3 Solar Direction Calculations

The direction of sunrise and sunset can be determined using the solar declination and hour angle. From our observer-centric perspective, we can calculate the azimuth angle (A) using:

$$\cos(A) = \frac{\sin(\delta) - \sin(\phi)\sin(h)}{\cos(\phi)\cos(h)} \tag{10}$$

where:

- δ is solar declination (-23.26°)
- ϕ is observer latitude (-53.15°)
- h is altitude (0° at horizon for sunrise/sunset)

7.4 Sunrise Direction

At sunrise:

$$\begin{aligned}\cos(A) &= \frac{\sin(-23.26) - \sin(-53.15)\sin(0)}{\cos(-53.15)\cos(0)} \\ &= \frac{-0.395}{0.602} \\ &= -0.656 \\ A &= 131.2\end{aligned}$$

Therefore, the sun rises at 131.2° (measured clockwise from North), or approximately 131° SE.

7.5 Sunset Direction

At sunset, the azimuth is symmetrical about the north-south line:

$$A_{sunset} = 360 - A_{sunrise} = 360 - 131.2 = 228.8 \tag{11}$$

Therefore, the sun sets at 228.8° (measured clockwise from North), or approximately 229° SW.

7.6 Solar Noon

The sun's maximum altitude occurs at local noon:

$$h_{max} = 90 - |\phi - \delta| = 90 - |-53.15 - (-23.26)| = 60.11 \tag{12}$$

8 Conclusions

This Flat Earth model successfully predicts sunrise (05:00 CLST) and sunset (22:00 CLST) using only flat planar trigonometry and local observations without even needing to know the physical location of the sun. The model demonstrates how celestial navigation can be performed without requiring complex spherical models, while still achieving accurate results that match empirical observations.